Correction for Mechanical Dead Space in the Calculation of Physiological Dead Space


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Abstract When physiological dead space (Vdp) is calculated for a patient who has alveolar dead space, e.g., after pulmonary vascular occlusion, less than the full volume of attached mechanical dead space (Vdm) appears in the measured dead space (Vdn). Under these conditions the traditional subtraction of Vdm from Vdn leads to underestimation of Vdp and can give a falsely small ratio of Vdp to tidal volume (VT) when, in fact, an abnormally large Vdp/VT exists. To make the proper correction for Vdm, two equations have been derived and validated with seven subjects having Vdp/VT from 0.29 to 0.87, using Vdn's from 120 to 322 ml. With only a small modification, these equations are suitable for routine clinical use and give Vdp/VT within 0.02 of that by the validated equations (32 of 33 comparisons). The fraction of Vdm subtracted from Vdn is the square of the ratio of effective alveolar to total alveolar ventilation and is never >1. This fraction is (Paco2/Paco2)2, where Paco2 and Paco2 are the mean partial pressures of expired alveolar and of arterial CO2 in the other equation this fraction is [PEco2/Paco2(VT - Vdan - Vdm)]2 where PEco2 is mixed expired Pco2 and Vdan is anatomical dead space. The second equation requires an estimated Vdan and is applicable when Paco2 is not measured or does not plateau (as in exercise).

Introduction Physiological dead space (Vdp) is becoming a more useful index of impaired perfusion of pulmonary vessels as normal values and conditions for Vdp become more precisely defined (1). When a normal adult breathes spontaneously at a normal tidal volume (VT), Vdp/VT is <0.45 and decreases with increased VT (1, 2). Vdp/VT may increase with age (1) and with a variety of parenchymal lung diseases. However, a very large ratio (e.g., 0.8 at a normal VT) or an increase of this ratio with the increased VT of exercise (2) supports the diagnosis of pulmonary vascular occlusion.

In the traditional calculation of Vdp, all of the mechanical dead space (Vdm) is subtracted from the measured dead space (Vdn). As given in Enghoff's modified Bohr equation (3),

\[ V_{dp} = V_{dn} - V_{dm} = \frac{P_{aco2} - P_{eco2}}{P_{aco2}} \times V_T - V_{dm} \] (1)

where Paco2 and Peo2 are the partial pressures of CO2 in arterial blood and mixed expired gas, respectively. Suwa and Bendixen have analyzed the change of Paco2 with added Vdm and have shown that total subtraction of Vdm can lead to underestimation of Vdp (4). This underestimation occurs whenever gas from Vdm is inspired into nonperfused alveoli where it has no direct effect on gas exchange with alveolar capillary blood.

We have derived and validated two equivalent equations for making the proper calculation and present evidence that shorter modified forms give, with precision Vdp at the VT of the measurement (VT)n.

1 Abbreviations used in this paper: Fv, fraction of ventilation to perfused alveoli; Paco2, mean concentration of CO2 in expired alveolar gas; Peo2, concentration of CO2 in mixed expired gas; Paco2, mean partial pressure of CO2 in expired gas; Paco2, mean partial pressure of CO2 (PCO2) in expired alveolar gas; Paco2, mean Pco2 in arterial blood; Peo2, Pco2 in mixed expired gas; r, correlation coefficient; Va, effective alveolar ventilation per breath; Vco2, volume of CO2 expired per breath; Vdm, alveolar dead space; Vdan, anatomical dead space; Vdn, total measured dead space; Vdm, mechanical dead space; Vdm, physiological dead space; VT, expired tidal volume.

2 Subscripts specific for this paper: n, value measured with Vdm attached; o, value without Vdm attached.
METHODS

Derivation. The volume of CO₂ expired in a breath (VCO₂) is given by

\[ VCO₂ = \frac{P_{E\text{CO}_2}V_T}{P_{A\text{CO}_2}} = \frac{P_{A\text{CO}_2}(V_T - V_{Dan} - V_{Dm})}{P_{A\text{CO}_2}V_T}, \]

where \( P_{E\text{CO}_2} \) and \( P_{A\text{CO}_2} \) are the mean concentrations of CO₂ in the mixed expired gas and in the expired alveolar gas, respectively, and \( V_{Dm} \) is anatomical dead space. Dividing by \( P_{A\text{CO}_2}V_T \) and converting concentrations to partial pressures:

\[ \frac{V_A}{V_T} = \frac{P_{E\text{CO}_2}}{P_{A\text{CO}_2}} = \frac{P_{A\text{CO}_2}(V_T - V_{Dan} - V_{Dm})}{P_{A\text{CO}_2}V_T}, \]

where \( V_A \) is measured effective alveolar ventilation per breath.

To derive equations for \( V_{DP} \), we have assumed that addition of \( V_{Dm} \) does not change the fraction of ventilation to perfused alveoli (\( f_p \)). Using the equation for \( f_p \) derived by Julian, Travis, Robin, and Crump (5), and denoting, respectively, the conditions with and without \( V_{Dm} \) by "n" and "o":

\[ f_p = \frac{P_{E\text{CO}_2}}{P_{A\text{CO}_2} - P_{A\text{CO}_2} + P_{E\text{CO}_2}} \]

\[ = \frac{P_{E\text{CO}_2}}{P_{A\text{CO}_2} - P_{A\text{CO}_2} + P_{E\text{CO}_2}} \]

Cross multiplying the middle and right sides of equation 4 and dividing by \( P_{A\text{CO}_2}P_{A\text{CO}_2} \):

\[ \frac{P_{E\text{CO}_2} - P_{E\text{CO}_2}P_{A\text{CO}_2}}{P_{A\text{CO}_2} - P_{A\text{CO}_2}P_{A\text{CO}_2}} \]

Substituting the left side of equation 3 (with appropriate subscripts) for the first term on each side of equation 5, and substituting the right side of equation 3 for \( P_{E\text{CO}_2}/P_{A\text{CO}_2} \) in the second term on each side of equation 5:

\[ V_A - \frac{P_{A\text{CO}_2}P_{A\text{CO}_2}(V_T - V_{Dann})}{P_{A\text{CO}_2}P_{A\text{CO}_2}V_T} \]

\[ = \frac{V_A}{V_T} = \frac{P_{A\text{CO}_2}P_{A\text{CO}_2}(V_T - V_{Dann} - V_{Dm})}{P_{A\text{CO}_2}P_{A\text{CO}_2}V_T} \]

Substituting \( (V_T - V_{Dn}) \) for \( V_A \) and \( (V_T - V_{Dp}) \) for \( V_{A:\text{co}} \):

\[ V_{DP} = \frac{V_T}{V_{DP}} \left[ V_T - \frac{P_{E\text{CO}_2}V_T}{P_{A\text{CO}_2}P_{A\text{CO}_2} - P_{E\text{CO}_2}V_T} \right] \]

Substitution for \( P_{A\text{CO}_2} \) from equation 3 gives an equivalent equation:

\[ V_{DP} = \frac{V_T}{V_{DP}} \left[ V_T - \frac{P_{E\text{CO}_2}V_T}{P_{A\text{CO}_2}(V_T - V_{Dann})} \right] \]

The validity of equations 7 and 8 can be tested by calculating \( V_{DP} \) for the same subjects with different \( V_{Dm} \)'s. If the assumption holds that \( f_p \) remains constant with added \( V_{Dm} \), then \( V_{DP} \) should remain constant.

In the routine measurement of dead space, \( V_{DP} \) is always calculated for \( V_T = V_{Tr} \). If \( V_{Dan} = V_{Daan} \), then equations 7 and 8 simplify to

\[ V_{DP} = V_{Dn} \left[ \frac{P_{A\text{CO}_2}P_{A\text{CO}_2}}{P_{A\text{CO}_2}P_{A\text{CO}_2}} \right] \cdot V_{Dm} \]

\[ V_{DP} = V_{Dn} \left[ \frac{P_{E\text{CO}_2}V_T}{P_{A\text{CO}_2}(V_T - V_{Dann})} \right] \cdot V_{Dm}. \]

Since equations 9 and 10 are equations 7 and 8 at the \( V_T \) and \( V_{Dm} \) of the measurement, experimental validation of equations 7 and 8 also validates 9 and 10.

If it can be shown that substitution of \( P_{A\text{CO}_2}/P_{A\text{CO}_2} \) for \( P_{A\text{CO}_2}/P_{A\text{CO}_2} \) (and the equivalent substitution in equation 10) result in very little error, then two good working equations are

\[ V_{DP} = V_{Dn} - \left[ \frac{P_{A\text{CO}_2}^2}{P_{A\text{CO}_2}P_{A\text{CO}_2}^2} \right] \cdot V_{Dm} \]

\[ V_{DP} = V_{Dn} - \left[ \frac{P_{E\text{CO}_2}V_T^2}{P_{A\text{CO}_2}(V_T - V_{Dann} - V_{Dm})} \right] \cdot V_{Dm}. \]

These equations, like equations 9 and 10, give \( V_{Dm} \) at \( V_{Tr} \); but they require data from only a single collection.

Measurements. \( V_{Dm} \) was measured in seven men chosen to represent a wide range of \( V_{Dm}/V_T \). The men sat upright and breathed spontaneously through a rubber mouth piece and Hans-Rudolph valve with a combined \( V_Dm \) of 120 ml. Additional measurements were made with pipes of varying lengths and internal diameter of 2 cm between the mouth piece and valve. Expired gas was collected in a 350 liter gasometer. The subjects breathed through each \( V_Dm \) for at least 8 min before each measurement was begun. Arterial blood was collected from an indwelling catheter during the middle minute of a 3 min gas collection. End-tidal CO₂ at the mouth and mixed expired CO₂ from the gasometer were measured with an infrared CO₂ meter. Alveolar CO₂ plateaued for each subject. Correction was made for the volume of gas lost through the CO₂ meter, and the total volume of expired gas was corrected to BTPS (body temperature, pressure, saturated with water) \( P_{CO_2}, P_{O_2} \), and pH were measured with an Instrumentation Laboratory blood gas analyzer (Institution Laboratory Inc., Lexington, Mass.). Both the infrared CO₂ meter and the CO₂ electrode were calibrated with the same gases, previously analyzed with the Scholander 0.5 cm³ gas analyzer. Duplicate measurements using the same \( V_Dm \) were made for six of the men. The reproducibility of \( V_Dm/V_T \) was within 0.02 for five men; the sixth (C. W.) had very disparate \( V_T \)'s with the same \( V_Dm \). Of the 41 collections, one deviated inexplicably for C. O. from all the other results (duplicate measurement with 120 ml \( V_Dm \) was discarded; however, duplicate measurements with 218 ml \( V_Dm \) were made with perfect agreement for \( V_Dm/V_T \).

Calculations. \( V_{DP} \) and \( V_{DP}/V_{Tr} \) were calculated for each collection using no correction (\( V_{Dm} \), the traditional correction (equation 1), and equations 9–12. Equations 7 and 8 were

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tested by using the VT and Pco2's of the initial condition (with 120 ml VDm) for the "o" condition (without VDm). Equations 8 and 10 include 120 ml VDm in the denominator of the ventilation ratio for the "o" condition. The last collection (instead of the first with the same 120 ml VDm) was used to calculate VDm/VT for C. W., because his initial VT far exceeded all subsequent VT's. The mean end-tidal Pco2 was used for PACO2, and VDm was estimated from ideal body weight (6, 7). Where \[ \frac{Paco2 VT}{Paco2 (VT - VDm - VDm)} \] was > 1 (C. O.), a value of 1 was used.

Regression lines (8) for VDP/VT on VDm/VT were calculated for each method and each subject. Comparisons of VDm/VT by equations 11 with 9 and by 12 with 10 were made, and correlation coefficients (r) were calculated (8). The initial

<table>
<thead>
<tr>
<th>Subject, diagnosis, age, height, estimated VDm</th>
<th>VDm (ml)</th>
<th>VT (ml)</th>
<th>Respiration rate per min</th>
<th>Pco2 (mm Hg)</th>
<th>PaCO2</th>
<th>PaCO2</th>
<th>VDm/VT (a)</th>
<th>VDp/VT (b)</th>
<th>VDp/Vp (c)</th>
<th>VDp/Vp (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C. O. Asthma in remission, 40 yr, 69 inches, 141 ml</td>
<td>120</td>
<td>473</td>
<td>19.0</td>
<td>18.0</td>
<td>36.9</td>
<td>38</td>
<td>0.53</td>
<td>0.27</td>
<td>0.29</td>
<td>0.27</td>
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<tr>
<td>C. W. Emphysema-bronchitis, 55 yr, 67 inches, 130 ml</td>
<td>120</td>
<td>891</td>
<td>10.7</td>
<td>22.4</td>
<td>37.8</td>
<td>40</td>
<td>0.44</td>
<td>0.31</td>
<td>0.42</td>
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<tr>
<td>J. R. Bronchitis-empysema, 70 yr, 66 inches, 120 ml</td>
<td>120</td>
<td>890</td>
<td>9.7</td>
<td>17.8</td>
<td>33.5</td>
<td>41</td>
<td>0.57</td>
<td>0.43</td>
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<tr>
<td>M. C. Emphysema-bronchitis, 56 yr, 69.5 inches, 144 ml</td>
<td>120</td>
<td>692</td>
<td>22.0</td>
<td>19.1</td>
<td>32.0</td>
<td>45</td>
<td>0.58</td>
<td>0.40</td>
<td>0.49</td>
<td>0.50</td>
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<tr>
<td>H. V. Bronchitis-empysema, 56 yr, 73 inches, 165 ml</td>
<td>120</td>
<td>1013</td>
<td>21.7</td>
<td>16.3</td>
<td>27.1</td>
<td>40</td>
<td>0.59</td>
<td>0.47</td>
<td>0.53</td>
<td>0.55</td>
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<tr>
<td>E. B. Emphysema-bronchitis, 56 yr, 69 inches, 141 ml</td>
<td>120</td>
<td>580</td>
<td>19.0</td>
<td>16.3</td>
<td>32.4</td>
<td>59</td>
<td>0.72</td>
<td>0.52</td>
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<tr>
<td>L. H. Embolism-empysema, 62 yr, 63.5 inches, 113 ml</td>
<td>120</td>
<td>478</td>
<td>33.0</td>
<td>7.1</td>
<td>16.8</td>
<td>59</td>
<td>0.88</td>
<td>0.63</td>
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</tbody>
</table>

TABLE I

VDP/VT with Added VDm Calculated by (a) No Correction for VDm, (b) Traditional Correction, (c) Equation 7, and (d) Equation 8

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measurements were not included in these correlations, because $\frac{P_{A}CO_{2}}{P_{A}CO_{2}}$ and $\frac{P_{H}CO_{2}}{P_{H}CO_{2}}$ (and their equivalents in equations 10 and 12) were identical.

**RESULTS**

Fig. 1 shows $\frac{V_{Dp}}{V_{T}}$ calculated by four methods for C. W. and L. H. The intercepts of the regression lines at $V_{Dm} = 0$ indicate the approximate $\frac{V_{Dp}}{V_{T}}$ ratios of the patients when breathing without $V_{Dm}$. As $\frac{V_{Dm}}{V_{Tn}}$ increases, $\frac{V_{Dn}}{V_{Tn}}$ and the traditional correction for C. W. appear to deviate equally from the intercept value of 0.44; neither gives the proper value. For L. H. (pulmonary embolism), the increase in $\frac{V_{Dn}}{V_{Tn}}$ is only 0.06 with 0.53 added $V_{Dm}/V_{Tn}$ (270 ml $V_{Dm}$); when the traditional correction is used, $\frac{V_{Dp}}{V_{T}}$ is 0.38, a gross underestimation of the 0.85 intercept. Table I (column b) shows for the group that underestimation of $\frac{V_{Dp}}{V_{T}}$ by the traditional correction increased as alveolar dead space increased.

Equations 7 and 8 give $V_{Dpo}$ at a constant $V_{T}$ (the assumed value for $V_{Tm}$). In Fig. 1 the results from equation 7 (rigorous correction using $P_{A}CO_{2}$) produce horizontal regression lines for both patients. This constancy in $\frac{V_{Dpo}}{V_{Tm}}$ is shown in Table I for each of the seven subjects and for both equations. For six of the seven men the intercepts of the regression lines for equations 7 and 8 at zero $V_{Dm}$ are within 0.02 of those for $\frac{V_{Dm}}{V_{Tm}}$.

Equations 9 and 10 calculate $\frac{V_{Dp}}{V_{T}}$ at the $V_{T}$ of the measurement (rather than at a constant $V_{T}$ as in equations 7 and 8). With added $V_{Dm}$ the subjects tended to increase $V_{T}$ slightly and $\frac{V_{Dp}}{V_{T}}$ by equations 9 and 10 decreases accordingly. The decrease for C. W. in Fig. 1 by equation 11 is essentially the same as by equation 9. This slight decrease represents a true decrease in $\frac{V_{Dp}}{V_{T}}$ with spontaneously increasing $V_{T}$ (1, 2). The encircled measurement in Fig. 1 ($V_{T}$ 373 ml greater than the repeat with the same $V_{Dm}$) is further evidence of this true decrease.

Values for $\frac{V_{Dp}}{V_{T}}$ by equations 11 and 12 agree within 0.02 of the values by the more rigorous equations 9 and 10 in 32 of 33 comparisons for both pairs of equations. The correlations are extremely high ($r = 0.998$ for each comparison).

**DISCUSSION**

Dead space in the original Bohr equation (9) or measured by Fowler's method (10) is $V_{Dm} + V_{Dm}$, i.e. mechanical dead space is simply an extension of $V_{Dm}$. However, physiological dead space includes alveolar dead space ($V_{Dn}$), which is a functional volume (11),

\[\text{Correction for Mechanical Dead Space} \]
defined as the difference between physiological and anatomical dead space (12). VTalveo is primarily caused by reduced perfusion of alveoli relative to their ventilation (13). If VDm were simply an extension of VDan during measurement of VDan, VDair would also equal VDm — VDan — VDm. The fact that VDp is underestimated when all of VDm is subtracted from VDb indicates that this is not so. As VDair increases an increasingly smaller fraction of dead space gas from VDm is inhaled into the perfused areas of the lung where it decreases effective ventilation (increases dead space). In the non-perfused alveoli VDb gas has little effect on dead space, i.e. effective ventilation cannot be decreased below 0. The nonperfused alveoli do contribute indirectly to gas exchange by exhaling part of the CO2 inhaled from VDm and VDm in the previous breath (12). When VDb is added, a smaller volume of this gas escapes to the outside, but its CO2 concentration is greater. Equations 7 and 8 take all these changes into account.

The nearly horizontal slopes of VDb/VT on VDm/VT and the agreement of the intercepts at VDm = 0 with those for VDb/VT indicate the validity of equations 7 and 8 and of the assumption used in their derivation (that fC does not change significantly with added VDb).

Equations 11 and 12 are good working equations to use with data from a single collection. As with the traditional correction for VDm, they apply only to the conditions (VTm, posture, spontaneous or assisted ventilation, etc.) of the measurement. The substitutions of PANC02/PA02 for PACE02/PA02, and the equivalent substitution using VDbm introduce little error as shown by the comparisons of equation 11 with 9 and 12 with 10.

Since the ratio within the brackets of equations 11 and 12 is the ratio of effective to total alveolar ventilation, it can never be > 1. When the calculated PACE02/PA02 in equation 11, or the equivalent ratio in equation 12 is > 1, simply subtract VDm from VDb.

Both equations 11 and 12 can be applied at large tidal volumes. However, if alveolar CO2 does not plateau, e.g., during the hyperventilation of exercise, then equation 12 using VDan is preferable. Any of several equations from the literature can be used for estimating VDan (14, 15). When comparing VDb/VT during exercise with rest, 3 ml should be added to the estimated VDb for every 100 ml over the subject’s resting VT (1, 16). When VT is very small, e.g., <300 ml for normal adults, correction for VDb is not dependable because alveolar ventilation begins before washout of VDan and VDb is complete (17).

Clinically, VDb/VT at normal tidal volumes gives a better index of alveolar dead space than do arterial-alveolar CO2 gradients when there is coexistent uneven ventilation. In addition, hyperventilation of exercise can be used to distinguish the normal decrease of VDb/VT from the increased VDb/VT accompanying pulmonary vascular occlusion (2). Since hyperventilation requires low resistance breathing valves having significant dead space, the proper correction for VDb is important.

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REFERENCES